Multiparty Session Types

as

Coherence Proofs

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Joint work with

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A Curry-Howard correspondence between Multiparty Session Types and Linear Logic.
From Linear Logic to Session Types, and back again

- Linear Logic [Girard, 87]

Curry-Howard correspondence! [Caires and Pfenning, 10] [Wadler, 12]
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  - Propositions as Session Types
Proofs
Propositions

\[ \vdash \Delta \]

where \( \Delta = A_1, \ldots, A_n \)
Classical Processes (CP) [Wadler, 12]

- Proofs as Processes
- Propositions as Session Types

\[
\vdash P \mid \Delta
\]

where \( \Delta = x_1 : A_1, \ldots, x_n : A_n \)

Read “Process \( P \) uses each channel \( x_i \) following protocol \( A_i \)”
Some rules (adapted from [Wadler, 12])
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\[
\frac{R \vdash \Sigma, y : A, x : B}{\overline{x} \ (y); \ R \vdash \Sigma, x : A \otimes \ B}
\]
Some rules (adapted from [Wadler, 12])

\[
\frac{P \vdash \Gamma, y:A}{x(y); (P \mid Q) \vdash \Gamma, \Delta, x:A \otimes B}
\quad \frac{Q \vdash \Delta, x:B}{x(y); (P \mid Q) \vdash \Gamma, \Delta, x:A \otimes B}
\]

\[
\frac{R \vdash \Sigma, y:A, x:B}{\bar{x}(y); R \vdash \Sigma, x:A \otimes B}
\quad \frac{R \vdash \Sigma, y:A, x:B}{\bar{x}(y); R \vdash \Sigma, x:A \otimes B}
\]
Some rules (adapted from [Wadler, 12])

\[
\frac{P \vdash \Gamma, y : A \quad Q \vdash \Delta, x : B}{x \,(y); \,(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B}
\quad \otimes
\]

\[
\frac{R \vdash \Sigma, y : A, x : B}{\overline{x} \,(y); \,R \vdash \Sigma, x : A \oslash B}
\quad \oslash
\]

\[
\frac{P \vdash \Gamma, x : A \quad Q \vdash \Delta, x : A^\perp}{(\nu x : A) \,(P \mid Q) \vdash \Gamma, \Delta}
\quad \text{Cut}
\]

where \( A^\perp \) is the “dual” of \( A \), e.g., \((A \otimes B)^\perp = A^\perp \oslash B^\perp\).
An example
An example

\[
\begin{align*}
P & \vdash \Gamma, y : A \\
Q & \vdash \Delta, x : B \\
\hline
x (y); (P \mid Q) & \vdash \Gamma, \Delta, x : A \otimes B
\end{align*}
\]
An example

\[
\begin{align*}
\frac{P \vdash \Gamma, y:A \quad Q \vdash \Delta, x:B}{x(y); (P \mid Q) \vdash \Gamma, \Delta, x:A \otimes B} \otimes \\
\frac{R \vdash \Sigma, x:A^\perp, y:B^\perp}{\bar{x}(y); R \vdash \Sigma, x:A^\perp \otimes B^\perp}
\end{align*}
\]
An example

\[
\begin{align*}
\frac{P \vdash \Gamma, y:A}{x(y); (P \mid \neg P) \vdash \Gamma, \Delta, x:A \otimes B} & \otimes \\
\frac{Q \vdash \Delta, x:B}{x(y); (P \mid \neg P) \vdash \Gamma, \Delta, x:A \otimes B} & \otimes \\
\frac{R \vdash \Sigma, y:A^\perp, x:B^\perp}{\bar{x}(y); R \vdash \Sigma, x:A^\perp \& B^\perp} & \otimes \\
\frac{(\nu x : A \otimes B) (x(y); (P \mid \neg P) | \Sigma)}{\bar{x}(y); R \vdash \Gamma, \Delta, \Sigma} & \otimes \\
\end{align*}
\]

because \((A \otimes B)^\perp = A^\perp \& B^\perp\).
Cut Elimination

In linear logic, cuts can always be eliminated from proofs.
Cut Elimination

\[
\begin{align*}
P \vdash & \quad \Gamma, y : A \\
Q \vdash & \quad \Delta, x : B \\
\overline{x} \ (y); \ (P \ | \ Q) \vdash & \quad \Gamma, \Delta, x : A \otimes B \\
\nu x \ (x \ (y); \ (P \ | \ Q)) \ | \ \overline{x} \ (y); \ R \vdash & \quad \Gamma, \Delta, \Sigma
\end{align*}
\]
Cut Elimination

\[
\begin{align*}
P \vdash \Gamma, y : A & \quad Q \vdash \Delta, x : B \\
\vdash \Gamma, \Delta, x : A \otimes B & \quad R \vdash \Sigma, y : A^\perp, x : B^\perp \\
(\nu x) (x (y); (P \mid Q)) & \quad \overline{x} (y); R \vdash \Sigma, x : A^\perp \otimes B^\perp \\
& \vdash \Gamma, \Delta, \Sigma
\end{align*}
\]

\$\times\$, \$\otimes\$
Cut Elimination

\[
\frac{\vdash \Gamma, y:A \quad \vdash \Delta, x:B}{\vdash \Gamma, \Delta, x:A \otimes B} \otimes \quad \frac{\vdash \Sigma, y:A^\perp, x:B^\perp}{\vdash \Sigma, x:A^\perp \otimes B^\perp} \quad \text{Cut}
\]

\[
\downarrow
\]

\[
\vdash \Delta, x:B
\]
**Cut Elimination**

\[
\frac{P \vdash \Gamma, y:A \quad Q \vdash \Delta, x:B}{x(y); (P \mid Q) \vdash \Gamma, \Delta, x:A \otimes B} \quad \otimes \quad \frac{R \vdash \Sigma, y:A\perp, x:B\perp}{x(y); R \vdash \Sigma, x:A\perp \otimes B\perp} \quad \otimes \\
(\nu x)(x(y); (P \mid Q) \mid x(y); R) \vdash \Gamma, \Delta, \Sigma \\
\downarrow \\
Q \vdash \Delta, x:B \quad R \vdash \Sigma, y:A\perp, x:B\perp}
\]
Cut Elimination

(\nu x \ (x (y); \ (P \ | \ Q) \ | \ \bar{x} \ (y); \ R) \vdash \Gamma, \Delta, \Sigma \quad \text{Cut}

Q \vdash \Delta, x : B \quad R \vdash \Sigma, y : A^\perp, x : B^\perp

(\nu x : B) (Q \ | \ R) \vdash \Delta, \Sigma, y : A^\perp \quad \text{Cut}
Cut Elimination

\[
\begin{align*}
P \vdash \Gamma, y : A & \quad Q \vdash \Delta, x : B \\
& \quad x (y); (P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B \\
& \quad (\nu x) (x (y); (P \mid Q) \mid x (y); R) \vdash \Gamma, \Delta, \Sigma \\
\end{align*}
\]

\[
\begin{align*}
R \vdash \Sigma, y : A^\perp, x : B^\perp \\
& \quad x (y); R \vdash \Sigma, x : A^\perp \otimes B^\perp \\
& \quad \Downarrow \\
& \quad R \vdash \Sigma, y : A^\perp, x : B^\perp \\
\end{align*}
\]

\[
\begin{align*}
P \vdash \Gamma, y : A & \quad Q \vdash \Delta, x : B \\
& \quad (\nu x : B) (Q \mid R) \vdash \Delta, \Sigma, y : A^\perp \\
\end{align*}
\]

Cut
Cut Elimination

\[
\frac{P \vdash \Gamma, y : A \quad Q \vdash \Delta, x : B}{x \, (y); \; (P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B} \otimes \quad \frac{R \vdash \Sigma, y : A \perp, x : B \perp}{\bar{x} \, (y); \; R \vdash \Sigma, x : A \perp \otimes B \perp} \quad \text{Cut}
\]

\[
\downarrow
\]

\[
\frac{Q \vdash \Delta, x : B \quad R \vdash \Sigma, y : A \perp, x : B \perp}{P \vdash \Gamma, y : A \quad (\nu x : B) \, (Q \mid R) \vdash \Delta, \Sigma, y : A \perp} \quad \text{Cut}
\]

\[
(\nu y : A) \, (P \mid (\nu x : B) \, (Q \mid R)) \vdash \Gamma, \Delta, \Sigma \quad \text{Cut}
\]
which corresponds to the typical reduction

\[(\nu x : A \otimes B) (x(y); (P \mid Q) \mid \bar{x}(y); R) \rightarrow (\nu y : A) (P \mid (\nu x : B) (Q \mid R))\]

where

\[
\frac{P \vdash \Gamma, y : A}{x(y); (P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B} \otimes \frac{R \vdash \Sigma, y : A^\perp, x : B^\perp}{\bar{x}(y); R \vdash \Sigma, x : A^\perp \otimes B^\perp}
\]

by Cut

\[
\frac{Q \vdash \Delta, x : B}{(\nu x : B) (Q \mid R) \vdash \Delta, \Sigma, y : A^\perp}
\]

by Cut

\[
\frac{P \vdash \Gamma, y : A}{(\nu y : A) (P \mid (\nu x : B) (Q \mid R)) \vdash \Gamma, \Delta, \Sigma}
\]

by Cut
A deep correspondence:
Curry-Howard Linear Logic $\leftrightarrow \pi$-calculus

A *deep* correspondence:

- Proofs *as* Processes
Curry-Howard Linear Logic $\leftrightarrow \pi$-calculus

A *deep* correspondence:

- Proofs *as* Processes
- Propositions *as* Session Types
Curry-Howard Linear Logic $\leftrightarrow$ $\pi$-calculus

A *deep* correspondence:

- Proofs *as* Processes
- Propositions *as* Session Types
- Cut Elimination *as* Communication
Benefits of the correspondence

- Canonicity, from the underlying re-appearing structure.
- Free results, e.g., deadlock-freedom from cut elimination.
- Reuse of well-understood logical tools. Examples:
  - Proof-carrying code [Pfenning et al., 11]
  - Typed translation from Functions to Processes [Toninho et al., 12]
  - Logical relations [Perez et al., 12]
  ...
Benefits of the correspondence

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  - Proof-carrying code [Pfenning et al., 11]
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  - ...
So far, the research flow has been Logic → Session Types.

But session types have a very active community (20 years).
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But session types have a very active community (20 years).
Where do the 20 years of results developed for session types go in this design?
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Can we import results from Session Types?
Session Types → Logic
Session Types does not check for taxes

buyer \( \bar{x} (\text{money}); \ x (\text{receipt}); \ P \) \\
| \\
seller \( x (\text{money}); \ \bar{y} (\text{taxes}); \ \bar{x} (\text{receipt}); \ Q \) \\
| \\
tax off. \( y (\text{taxes}); \ R \)
I can forget paying my taxes!

\[
\begin{align*}
\text{buyer} & \quad \overline{x} \ (money); \ x \ (receipt) ; \ P \\
\text{\quad |} \\
\text{seller} & \quad x \ (money); \ \overline{x} \ (receipt) ; \ Q
\end{align*}
\]
Multiparty Session Types [Honda et al., 08]

buyer \[\overline{x} \ (money); \ x \ (receipt); \ P\]
| \[\]

seller \[x \ (money); \ \overline{y} \ (taxes); \ \overline{x} \ (receipt); \ Q\]
| \[\]

tax off. \[y \ (taxes); \ R\]
Multiparty Session Types [Honda et al., 08]

<table>
<thead>
<tr>
<th>buyer</th>
<th>$\overline{x} \ (\text{money}); \ x \ (\text{receipt}); \ P$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>seller</td>
<td>$x \ (\text{money}); \ \overline{x} \ (\text{taxes}); \ \overline{x} \ (\text{receipt}); \ Q$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>tax off.</td>
<td>$x \ (\text{taxes}); \ R$</td>
</tr>
</tbody>
</table>
Multiparty Session Types [Honda et al., 08]

buyer  \( x^{BS}(\text{money}); x^{BS}(\text{receipt}); P \)  
        |  
seller  \( x^{SB}(\text{money}); x^{ST}(\text{taxes}); x^{SB}(\text{receipt}); Q \)  
        |  
tax off. \( x^{TS}(\text{taxes}); R \)
Multiparty Session Types [Honda et al., 08]

buyer \( \overline{x}^{BS}(money); \ x^{BS}(receipt); \ P \)

\[ \]

seller \( x^{SB}(money); \ \overline{x}^{ST}(taxes); \ \overline{x}^{SB}(receipt); \ Q \)

\[ \]

tax off. \( x^{TS}(taxes); \ R \)

The type of \( x \) is a global type:

\( B \rightarrow S : \langle \rangle; \ S \rightarrow T : \langle \rangle; \ S \rightarrow B : \langle \rangle \)
Type checking in MPSTs

From the global type

\[ B \rightarrow S : \langle \rangle; \ S \rightarrow T : \langle \rangle; \ S \rightarrow B : \langle \rangle \]
Type checking in MPSTs

From the global type

\[ B \rightarrow S : \langle \rangle; \ S \rightarrow T : \langle \rangle; \ S \rightarrow B : \langle \rangle \]

project the \textit{local type} for each role:

- role \( B \) : send \( S \); recv \( S \)
- role \( S \) : recv \( B \); send \( T \); send \( B \)
- role \( T \) : recv \( S \)
So Multiparty Session Types are not based on duality!
Rather, the compositionality principle is called coherence:

**Definition (Coherence)**
A set of local types is coherent if they can all be projected from one global type.

*Can we really adapt linear logic to this radical change?*
Local typing just requires repainting the correspondence!
From Duality to Coherence

Local typing just requires repainting the correspondence!

\[
\frac{P \vdash \Gamma, y:A \quad Q \vdash \Delta, x:B}{x \ (y); \ (P \mid Q) \vdash \Gamma, \Delta, x:A \otimes B} \quad \otimes
\]
Local typing just requires repainting the correspondence!

\[
\frac{P \vdash \Gamma, y^p : A \quad Q \vdash \Delta, x^p : B}{x^{pq}(y); (P \mid P) \vdash \Gamma, \Delta, x^p : A \otimes^q B}
\]
Local typing just requires repainting the correspondence!

\[
\frac{P \vdash \Gamma, y^p : A \quad Q \vdash \Delta, x^p : B}{x^p q(y); (P \mid Q) \vdash \Gamma, \Delta, x^p : A \otimes q B} \otimes \quad \frac{R \vdash \Sigma, y : A, x : B}{\overline{x} (y); R \vdash \Sigma, x : A \& B}
\]
Local typing just requires repainting the correspondence!

\[
\frac{P \vdash \Gamma, y^p : A \quad Q \vdash \Delta, x^p : B}{x^p q(y) ; (P \mid Q) \vdash \Gamma \amp \Delta, x^p : A \otimes q B}
\quad \otimes

\frac{R \vdash \Sigma, y^q : A, x^q : B}{\bar{x} q p(y) ; R \vdash \Sigma, x^q : A \otimes^p B}
\]
Typing Buyer-Seller-Taxes

buyer $\overline{x}^{BS}(\text{money}); x^{BS}(\text{receipt}); P$

seller $x^{SB}(\text{money}); \overline{x}^{ST}(\text{taxes}); \overline{x}^{SB}(\text{receipt}); Q$

tax off. $x^{TS}(\text{taxes}); R$
Typing Buyer-Seller-Taxes

buyer
\[ \overline{x}^{BS}(money); \overline{x}^{BS}(receipt); P \]

\[ \underline{x}^{SB}(money); \overline{x}^{ST}(taxes); \overline{x}^{SB}(receipt); Q \]

\[ \text{tax off.} \quad x^{TS}(taxes); R \]

\[ \Gamma, x^B: \bot \otimes^S (1 \otimes^S A) \]
Typing Buyer-Seller-Taxes

buyer \[ x^{BS}(money); \ x^{BS}(receipt); \ P \]

\[ \vdash \Gamma, x^B: \bot \otimes^S (1 \otimes^S A) \]

seller \[ x^{SB}(money); \ x^{ST}(taxes); \ x^{SB}(receipt); \ Q \]

\[ \vdash \Delta, x^S: 1 \otimes^B (\bot \otimes^T (\bot \otimes^B B)) \]
Typing Buyer-Seller-Taxes

\[
\begin{align*}
\text{buyer} & \quad x^{BS}(money); x^{BS}(receipt); P \\
\text{seller} & \quad x^{SB}(money); x^{ST}(taxes); x^{SB}(receipt); Q \\
\text{tax off.} & \quad x^{TS}(taxes); R
\end{align*}
\]

\[
\begin{align*}
\text{buyer} & \quad \vdash \Gamma, x^B : \bot \otimes^S (1 \otimes^S A) \\
\text{seller} & \quad \vdash \Delta, x^S : 1 \otimes^B (\bot \otimes^T (\bot \otimes^B B)) \\
\text{tax off.} & \quad \vdash \Sigma, x^T : 1 \otimes^S C
\end{align*}
\]
Composing Multiparty Processes

\[
\begin{align*}
\text{buyer} & \vdash \Gamma, x^B : \perp \otimes^S (1 \otimes^S A) \\
\text{seller} & \vdash \Delta, x^S : 1 \otimes^B (\perp \otimes^T (\perp \otimes^B B)) \\
\text{tax off.} & \vdash \Sigma, x^T : 1 \otimes^S C
\end{align*}
\]
Composing Multiparty Processes

buyer \vdash \Gamma, x^B : \bot \otimes^S (1 \otimes^S A)

seller \vdash \Delta, x^S : 1 \otimes^B (\bot \otimes^T (\bot \otimes^B B))

tax off. \vdash \Sigma, x^T : 1 \otimes^S C

How can we compose them?
Composing Multiparty Processes

buyer \vdash \Gamma, x^B : \perp \otimes S (1 \otimes S A)

seller \vdash \Delta, x^S : 1 \otimes B (\perp \otimes T (\perp \otimes B B))

tax off. \vdash \Sigma, x^T : 1 \otimes S C

How can we compose them? First attempt:

\[
\frac{P_i \vdash \Gamma_i, x^{p_i} : A_i \quad \exists G \text{ s.t. } \text{proj}(G) = \{p_i : A_i\}_i}{(\nu x : G) (\prod_i P_i) \vdash \{\Gamma_i\}_i} \quad \text{MCut}
\]
First attempt:

\[
\frac{P_i \vdash \Gamma_i, x^{p_i} : A_i \quad \exists G \text{ s.t. } \text{proj}(G) = \{p_i : A_i\}_i}{(\nu x : G) \left( \prod_i P_i \right) \vdash \{\Gamma_i\}_i} \quad \text{MCut}
\]

Two problems with that condition:
First attempt:

\[
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\]

\[
(\nu x : G) (\prod_i P_i) \vdash \{\Gamma_i\}_i
\]

MCut

Two problems with that condition:

▶ it does not tell us how to prove it;
First attempt:

\[ P_i \vdash \Gamma_i, x^{p_i} : A_i \quad \exists G \text{ s.t. } \text{proj}(G) = \{p_i : A_i\}_i \]

\[ (\nu x : G) \left( \prod_i P_i \right) \vdash \{\Gamma_i\}_i \quad \text{MCut} \]

Two problems with that condition:

- it does not tell us how to prove it;
- it does not tell us why the composition is safe.
We propose to treat coherence as a proof system:

\[ P_i \vdash \Gamma_i, x^{p_i} : A_i \quad G \models \{ p_i : A_i \}_i \]

\[ (\nu x : G) (\prod_i P_i) \vdash \{ \Gamma_i \}_i \quad \text{MCut} \]
Coherence is simple

Here are all the four rules:

\[
\begin{align*}
G & \models \Theta, \ p : B, \ \{q_i : D_i\}_i \\
G' & \models \ p : A, \ \{q_i : C_i\}_i \\
\end{align*}
\]

\[
p \to \tilde{q} : \langle G' \rangle; G \models \Theta, \ p : A \otimes \tilde{q} B, \ \{q_i : C_i \otimes^p D_i\}_i
\]

\[
\text{end}^{p\tilde{q}} \models p : \bot, \ q_1 : 1, \ldots, q_n : 1
\]

\[
G_1 \models \Theta, \ p : A, \ \{q_i : C_i\}_i \\
G_2 \models \Theta, \ p : B, \ \{q_i : D_i\}_i
\]

\[
p \to \tilde{q} : \& (G_1, G_2) \models \Theta, \ p : A \oplus \tilde{q} B, \ \{q_i : C_i \&^p D_i\}_i
\]

\[
G \models p : A, \ \{q_i : B_i\}_i
\]

\[
?p \to !\tilde{q} : \langle G \rangle \models p : ?A, \ \{q_i : !B_i\}_i
\]
Coherence looks right

- Isomorphism between well-formed global types and coherence proofs: Global Types as Coherence Proofs!
- We know how to prove the condition $G \models \{p_i : A_i\}_i$ now: just do a proof.
Coherence looks right

- Isomorphism between well-formed global types and coherence proofs: Global Types as Coherence Proofs!
- We know how to prove the condition $G \models \{p_i : A_i\}_i$ now: just do a proof.
- But most importantly...
Coherence looks right

- Isomorphism between well-formed global types and coherence proofs: Global Types as Coherence Proofs!
- We know how to prove the condition \( G \models \{ p_i : A_i \}_i \) now: just do a proof.
- But most importantly...
  - We know why it works: Cut Elimination!
A communication

\[(\nu x : p \rightarrow \tilde{q} : \langle G' \rangle ; G) \left( \prod_i x^{q_i} p(y); (P_i | Q_i) | \bar{x}^{p\tilde{q}}(y); R | \prod_j P_j \right)\]
A communication

$$(\nu x : p \rightarrow \tilde{q} : \langle G' \rangle ; G) \left( \prod_i x^{q_i} p(y); (P_i \mid Q_i) \mid \bar{x}^{p\tilde{q}}(y); R \mid \prod_j P_j \right)$$

$$\rightarrow \quad (\nu y : G') \left( \prod_i P_i \mid (\nu x : G) (\prod_i Q_i \mid R \mid \prod_j P_j) \right)$$
Results

Session fidelity: reductions follow the protocols.

Cut Elimination, and hence deadlock-freedom.
Results

- **Session fidelity**: reductions follow the protocols.
Results

- **Session fidelity**: reductions follow the protocols.
- **Cut Elimination**, and hence deadlock-freedom.
More on coherence

Projection: a global type yields a set of corresponding local types, by isomorphism with coherence proofs.

Extraction: a proof search for coherence extracts the global type that some local types follow.
More on coherence

- **Projection**: a global type yields a set of corresponding local types, by isomorphism with coherence proofs.
More on coherence

- **Projection**: a global type yields a set of corresponding local types, by isomorphism with coherence proofs.
- **Extraction**: a proof search for coherence extracts the global type that some local types follow.
In the paper

- All the rules.
- More nice properties.
- Examples with multiple sessions.
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Conclusions

- Curry-Howard goes both ways.
Conclusions

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- What new things will arise?
Thank you!
Thank you!

Questions?